

Level - 3	JEE Advanced++ Pattern/Solutions
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147. (i) $h_f = h\{1 + \alpha_c \Delta\theta\}$, $A_f = A\{1 + 2\alpha_c \Delta\theta\}$, $V_f = Ah\{1 + 3\alpha_c \Delta\theta\}$ (ii) $V'_f = Ah_1(1 + \gamma_L \Delta\theta)$

(iii) $h_{1f} = h_1\{1 + \gamma_L \Delta\theta\}$ (iv) $h_{1f} = h_1(1 - 2\alpha_c \Delta\theta)$ (v) $3h\alpha_c = h_1\gamma_L$

(i) New height $h_f = h\{1 + \alpha_c \Delta\theta\}$

New area of cross section $A_f = A\{1 + 2\alpha_c \Delta\theta\}$

New volume of container $V_f = Ah\{1 + 3\alpha_c \Delta\theta\}$

(ii) New volume of liquid $V'_f = V_0(1 + \gamma_L \Delta\theta) = Ah_1(1 + \gamma_L \Delta\theta)$

(iii) The height of liquid level when expansion of container is neglected

$$h_{1f} = \frac{V'_f}{A} = \frac{Ah_1(1 + \gamma_L \Delta\theta)}{A} \Rightarrow h_{1f} = h_1\{1 + \gamma_L \Delta\theta\}$$

(iv) The area of container will increase, the area of container at this temperature will be

$$A_f = A\{1 + 2\alpha_c \Delta\theta\}$$

As liquid does not expand the volume of the liquid will be as initial volume $= Ah_1$.

Hence height of the liquid column will be

$$h_{1f} = \frac{Ah_1}{A_f} = \frac{Ah_1}{A(1 + 2\alpha_c \Delta\theta)} \Rightarrow h_{1f} = h_1(1 - 2\alpha_c \Delta\theta)$$

(v) The initial volume of container above the liquid $V_1 = Ah - Ah_1$

Final volume of container above the liquid $V_2 = Ah(1 + 3\alpha_c \Delta\theta) - Ah_1(1 + \gamma_L \Delta\theta)$

If volume of container above liquid remains constant, $V_2 = V_1$

$$[Ah(1 + 3\alpha_c \Delta\theta) - Ah_1(1 + \gamma_L \Delta\theta)] = [Ah - Ah_1]$$

Which gives $3h\alpha_c = h_1\gamma_L$

148. $(2 \times 10^{-4} / ^\circ\text{C})$

We start from the liquid surface of column A, as we move along the tube to liquid surface in column D. When we move down, pressure increase and decreases as we move up.

$$P_0 + \rho_A gh_A - \rho_B gh_B + \rho_C gh_C - \rho_D gh_D = P_0$$

$$\rho_A gh_A - \rho_B gh_B + \rho_C gh_C - \rho_D gh_D = 0$$

as $\rho_A = \rho_C = \rho_{95^\circ\text{C}}$

$$\rho_B = \rho_D = \rho_{5^\circ\text{C}} \Rightarrow \rho_{95^\circ\text{C}}(h_A + h_C) = \rho_{5^\circ\text{C}}(h_B + h_D)$$

$$\frac{\rho_{5^\circ\text{C}}}{\rho_{95^\circ\text{C}}} = \frac{h_A + h_C}{h_B + h_D} = \frac{52.8 + 49}{49 + 51} = 1.018$$

as $\rho_{95^\circ\text{C}} = \frac{\rho_{5^\circ\text{C}}}{[1 + \gamma(90^\circ\text{C})]}$

$$\frac{\rho_{5^\circ\text{C}}}{\rho_{95^\circ\text{C}}} = [1 + \gamma(90^\circ\text{C})] = 1.018$$

Hence $\gamma = \left(\frac{1.018 - 1}{90^\circ\text{C}} \right) = 2 \times 10^{-4} / ^\circ\text{C}$

149. $\left(\frac{k\pi r_1 r_2 (T_2 - T_1)}{L} \right)$

Here $r = r_1 + \left(\frac{r_2 - r_1}{L} \right) x$

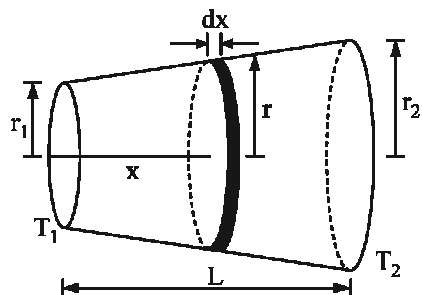
As shown in figure we consider an elemental disc at a distance x from left face. Then thermal resistance of this elemental disc is given as

$$dR_{th} = \frac{1}{k} \cdot \frac{dx}{\pi \left(r_1 + \frac{r_2 - r_1}{L} x \right)^2}$$

Total thermal resistance of frustum is

$$\begin{aligned} R_{th} &= \int dR_{th} = \frac{1}{k\pi} \int_0^L \frac{dx}{\left(r_1 + \frac{r_2 - r_1}{L} x \right)^2} \\ &= \frac{L}{k\pi(r_2 - r_1)} \left[-\frac{1}{\left(r_1 + \frac{r_2 - r_1}{L} x \right)} \right]_0^L = \frac{L}{k\pi(r_2 - r_1)} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{L}{k\pi r_1 r_2} \end{aligned}$$

Thus heat current through frustum is $\frac{dQ}{dt} = \frac{T_2 - T_1}{R_{th}} = \frac{k\pi r_1 r_2 (T_2 - T_1)}{L}$



150. $\left(T = T_1 - \frac{(r^2 - R_1^2)(T_1 - T_2)}{R_2^2 - R_1^2} \right)$

Let heat transferred per second is H , then

$$H = -K(2\pi rl) \frac{dT}{dr}$$

or $H = -2\pi lr \frac{\alpha}{r^2} \frac{dT}{dr} = -2\pi l \frac{\alpha}{r} \frac{dT}{dr}$

$$\int_{R_1}^{R_2} r dr = -\frac{2\pi l \alpha}{H} \int_{T_1}^{T_2} dT$$

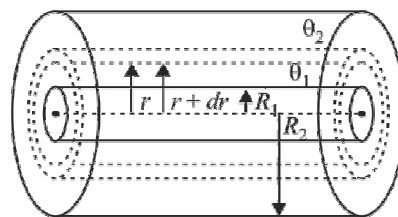
$$H = \frac{4\pi l \alpha (T_1 - T_2)}{(R_2^2 - R_1^2)} \quad \dots (1)$$

$$\int_{R_1}^r r dr = \frac{-2\pi l \alpha}{H} \int_{T_1}^T dT, \quad \frac{r^2 - R_1^2}{2} = \frac{2\pi l \alpha}{H} (T_1 - T)$$

$$T - T_1 = \frac{(r^2 - R_1^2)H}{4\pi l \alpha} \quad \text{or} \quad T = T_1 - \frac{(r^2 - R_1^2)H}{4\pi l \alpha} \quad \dots (2)$$

From eqns. (1) and (2), $T = T_1 - \frac{(r^2 - R_1^2)(T_1 - T_2)}{R_2^2 - R_1^2}$

Assume that heat absorbed by sphere is negligible. Take specific heat of water $c = 4.2 \text{ kJ/kg K}$, latent heat of vaporization $L_v = 2268 \text{ kJ/kg}$.



151. (i) 9 min 39 s (ii) 54 min 49 s

(i) Let T be the instantaneous temperature of the water. The rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{T_0 - T}{R_{th}} \quad \dots (i)$$

Where R_{th} is the equivalent thermal resistance of the hollow sphere. Consider a thin spherical sheet of radius r and thickness dr as shown in the figure. Then

$$dR_{th} = \frac{dr}{K(4\pi r^2)}$$

$$R_{th} = \frac{1}{4\pi K} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{R_2 - R_1}{4\pi K R_1 R_2}$$

For $R_1 = 0.25 \text{ m}$; $R_2 = 0.50 \text{ m}$; $K = \frac{10^2}{4\pi} \text{ W/m-K s}$

$$R = \frac{0.5 - 0.25}{4\pi \left(\frac{100}{4\pi}\right) (0.50)(0.25)} = 2 \times 10^{-2} \text{ K/W}$$

Heat required to raise the temperature of water by dT is given by $dQ = mc dT$. Substituting the value of dQ in equation (1), we get

$$mc \frac{dT}{dt} = \frac{T_0 - T}{R} \quad \text{or} \quad mcR \int_0^T \frac{dT}{T_0 - T} = \int_0^t dt$$

$$\text{or} \quad mcR \ln \left| \frac{T_0}{T_0 - T} \right| = t$$

On substituting numerical values, we get t

$$c = 4.2 \times 10^3 \text{ J/kg}; m = \rho V = (10)^3 \times \left[\frac{4}{3} \pi (0.25)^3 \right] = 65.45 \text{ kg}$$

$$R = 2 \times 10^{-2} \text{ K/W}$$

$$T_0 = 1000^\circ\text{C}; T = 100^\circ\text{C}$$

$$t = (65.45)(4.2 \times 10^3)(2 \times 10^{-2}) \ln \left| \frac{1000}{1000 - 100} \right| = 576 \text{ s} = 9 \text{ min } 36 \text{ s}$$

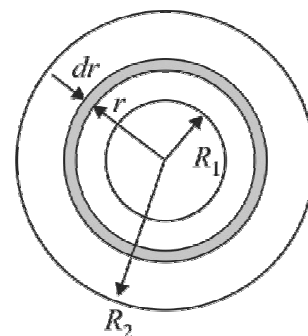
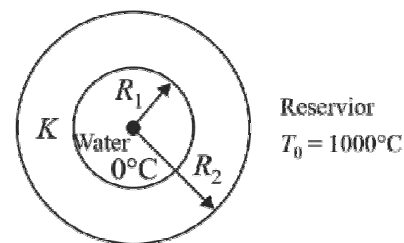
(ii) Heat required to convert 65.45 kg of water into steam is

$$Q = mL_v = (65.45)(2268 \times 10^3) = 1.48 \times 10^8 \text{ J}$$

The rate of heat flow is

$$\frac{dQ}{dt} = \frac{\Delta T}{R} = \frac{1000 - 100}{2 \times 10^{-2}} = 4.5 \times 10^4 \text{ W}$$

$$\therefore \text{Time required is } t = \frac{1.48 \times 10^8}{4.5 \times 10^4} = 3289 \text{ s} = 54 \text{ min } 49 \text{ s}$$



152.
$$\left(T = T_0 + \frac{bH}{AK} \left[1 - e^{-\left(\frac{AK}{bc} t \right)} \right] \right)$$

Heat conducted through metallic slab,

$$\frac{dQ}{dt} = \frac{KA(T - T_0)}{b}$$

Heat gain by water through heater = H

Net rate at which heat is gained by water,

$$\frac{dQ}{dt} = H - \frac{KA}{b}(T - T_0), \quad c \frac{dT}{dt} = H - \frac{KA}{b}(T - T_0)$$

$$\int_{T_0}^T \frac{cdT}{H - \frac{KA}{b}(T - T_0)} = \int_0^t dt, \quad c \left(\frac{-b}{AK} \right) \left| \ln \left\{ H - \frac{AK(T - T_0)}{b} \right\} \right|_{T_0}^T = t$$

$$-\frac{bc}{AK} \ln \left[\frac{H - \frac{AK}{b}(T - T_0)}{H} \right] = t, \quad H - \frac{AK(T - T_0)}{b} = He^{-\left(\frac{AKt}{bc} \right)}$$

$$\frac{AK(T - T_0)}{b} = H \left[1 - e^{-\left(\frac{AK}{bc} t \right)} \right], \quad T = T_0 + \frac{bH}{AK} \left[1 - e^{-\left(\frac{AK}{bc} t \right)} \right]$$